n. 220

$$y = \frac{2x-1}{x+2} \qquad D = \mathbb{R} - \{-2\}$$

$f: \mathbb{R}-\{-2\} \to \mathbb{R}-\{2\}$

$f^{-1}: \mathbb{R}-\{2\} \to \mathbb{R}-\{-2\}$

- Iniettiva

$\forall x_1, x_2 \in \text{dom} f, \; x_1 \neq x_2 \Rightarrow$

$\Rightarrow f(x_1) \neq f(x_2)$

$f(x_1) = f(x_2) \Rightarrow x_1 = x_2$

$$\frac{2x_1 - 1}{x_1 + 2} = \frac{2x_2 - 1}{x_2 + 2}$$

$$\cancel{2x_1x_2} + 4x_1 - x_2 - \cancel{2} = \cancel{2x_1x_2} - x_1 + 4x_2 - \cancel{2}$$

$$5x_1 = 5x_2 \Rightarrow x_1 = x_2$$

iniettiva

- Suriettiva

$\forall y \in \mathbb{R} \quad \exists x \in \mathbb{R} \quad : f(x) = y$

$$y = \frac{2x-1}{x+2} \Rightarrow xy + 2y = 2x - 1$$

$$xy - 2x = -2y - 1$$

$$x(y-2) = -2y - 1$$

$$x = \frac{-2y-1}{y-2} \qquad \mathbb{R} - \{2\}$$

$$\rightarrow y = \frac{-2x-1}{x-2}$$

$$y = \frac{1}{x+2} \Rightarrow xy + 2y = 1$$

$$xy = 1 - 2y \qquad x = \frac{1-2y}{y}$$

$$\mathbb{R} - \{0\}$$

$$y = \frac{x^2 - 1}{x + 2}$$

- Iniettiva: $\forall x_1, x_2 \in \mathbb{R},\ x_1 \neq x_2 \Rightarrow f(x_1) \neq f(x_2)$

$$\frac{x_1^2 - 1}{x_1 + 2} = \frac{x_2^2 - 1}{x_2 + 2}$$

$$x_1^2 x_2 + 2x_1^2 - x_2 - \cancel{1} = x_1 x_2^2 - x_1 + 2x_2^2 - \cancel{1}$$

$$x_1^2 x_2 - x_1 x_2^2 = x_2 - x_1 + 2x_2^2 - 2x_1^2$$

$$x_1 x_2 (x_1 - x_2) = x_2 - x_1 + 2(x_2 - x_1)(x_2 + x_1)$$

$$x_1 x_2 \cancel{(x_1 - x_2)} = \cancel{(x_2 - x_1)}(1 + 2x_2 + 2x_1)$$

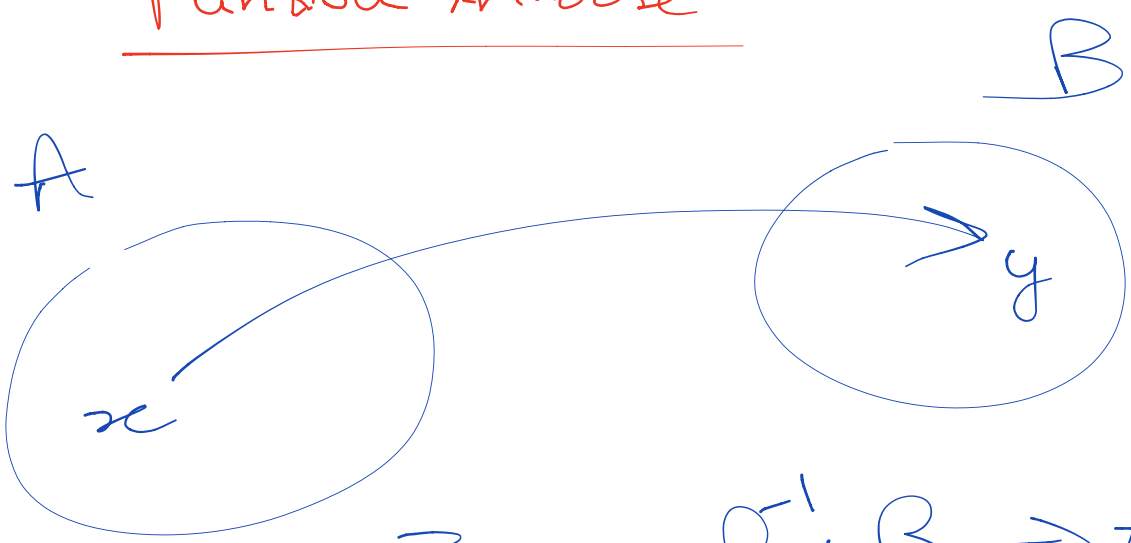
$x_1 \neq x_2$

$$y = x^2 - 1$$

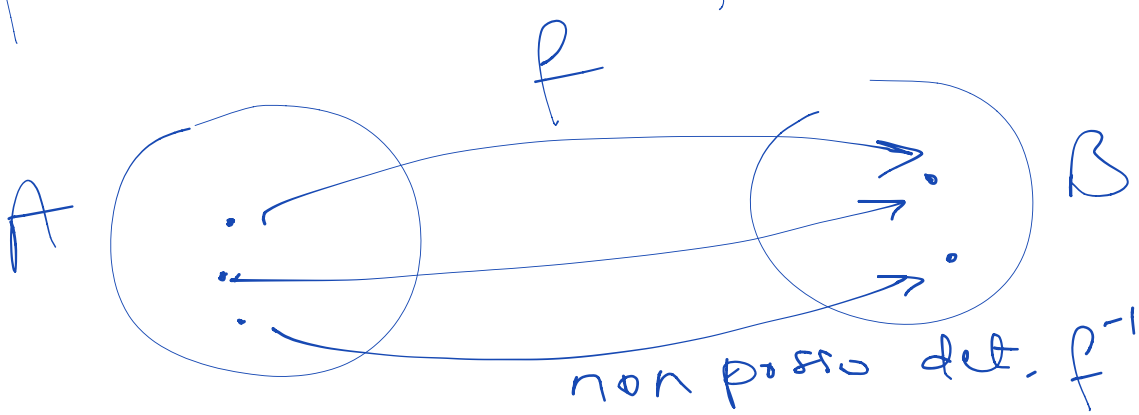
$$x_1^2 - \cancel{1} = x_2^2 - \cancel{1} \Rightarrow x_1^2 = x_2^2$$

$$x_1 = \pm x_2 \quad \text{non iniettiva}$$

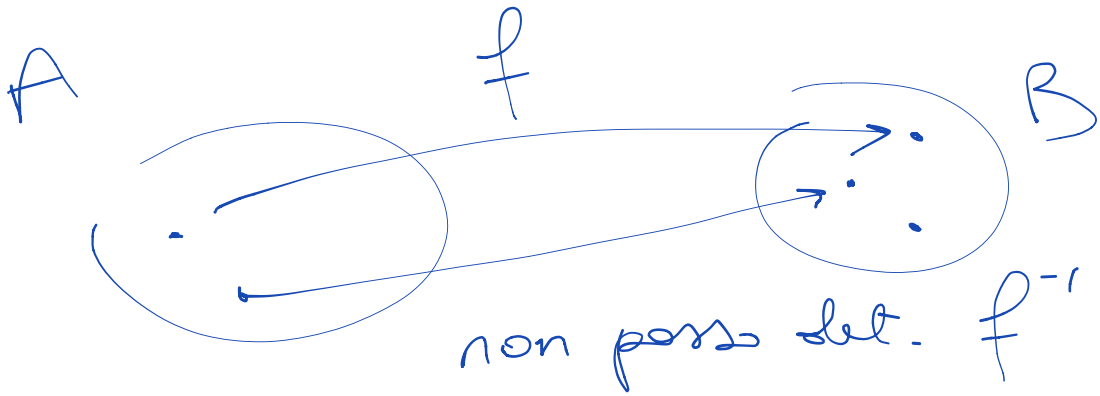
Funzioni inverse



$f : A \to B$ $\qquad$ $f^{-1} : B \to A$

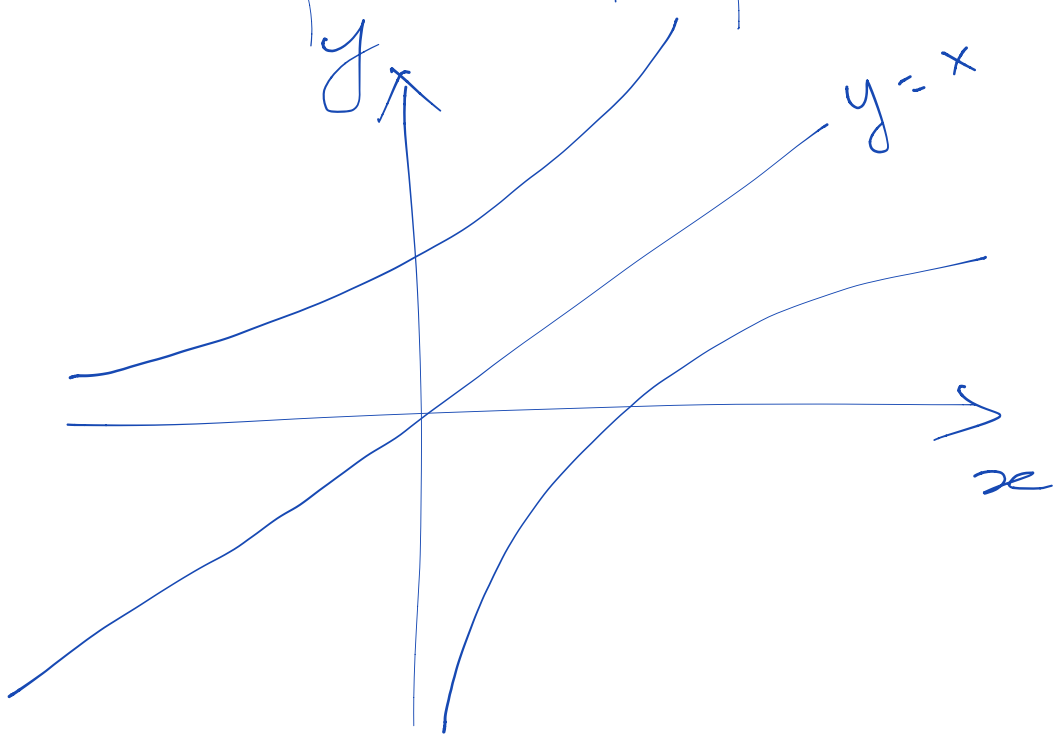


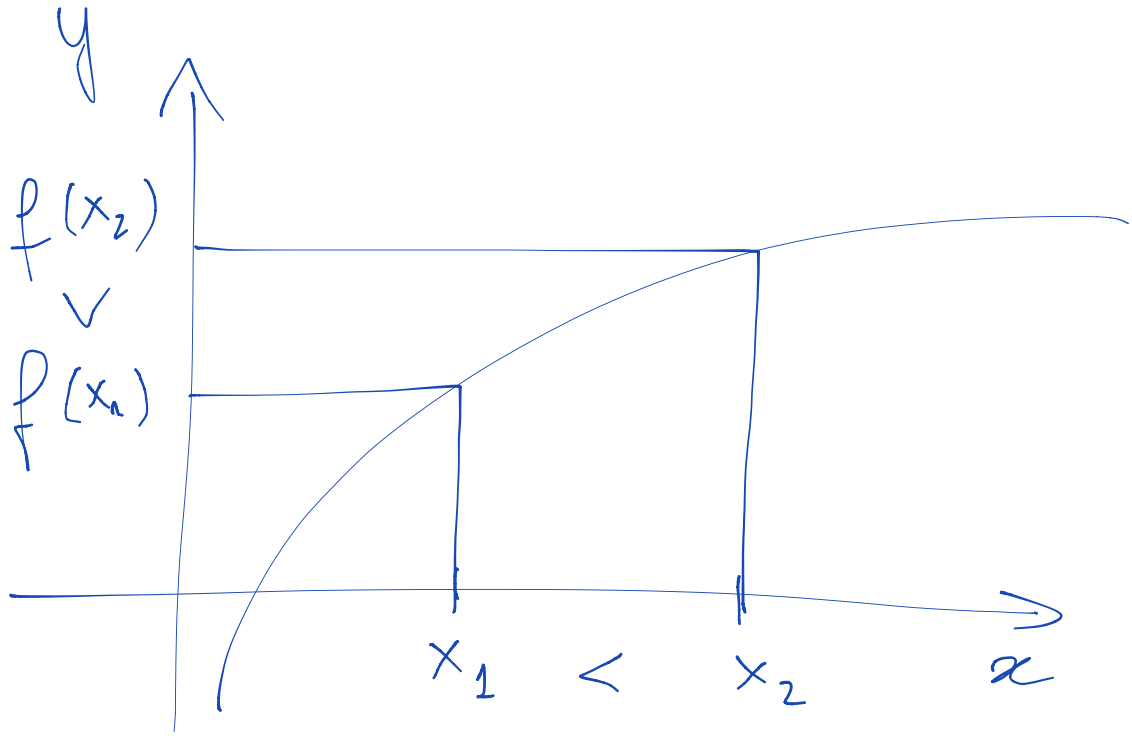
non posso def. f^{-1}



non posso def. f^{-1}

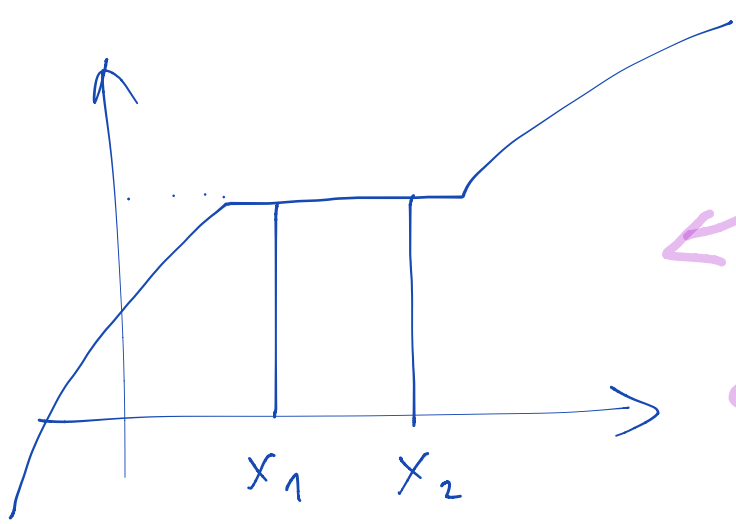
$$\operatorname{Im} f = \operatorname{cod} f = f(A) \subseteq B$$



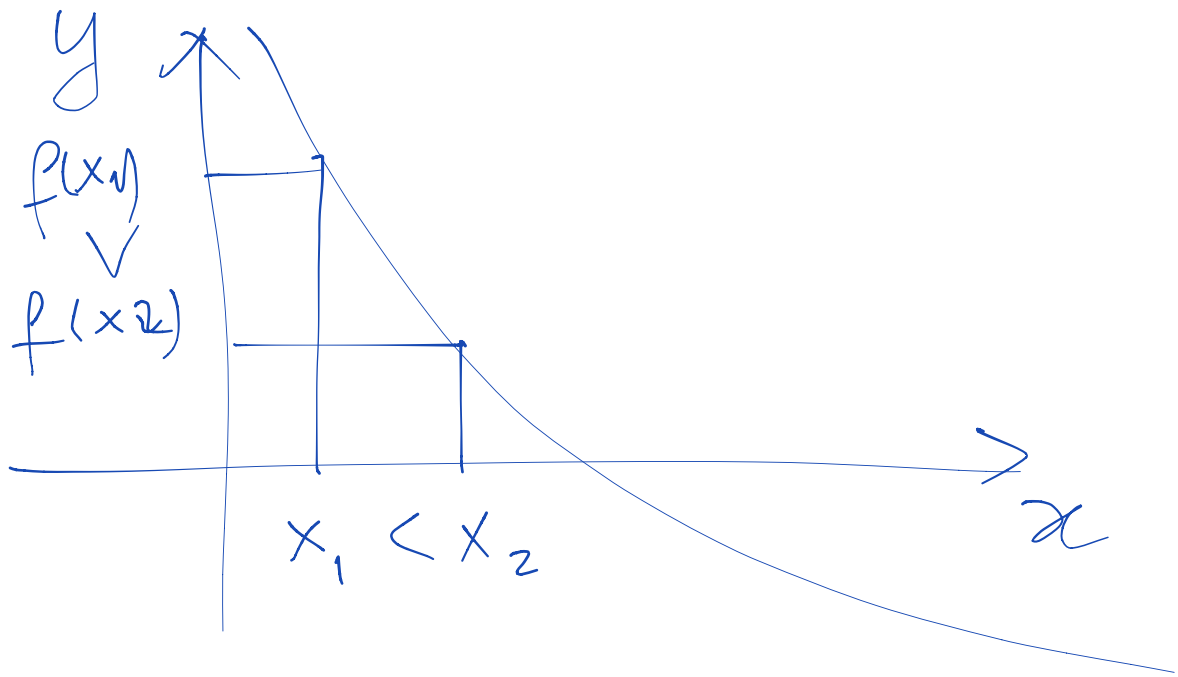


f è crescente se

$\forall x_1, x_2 \in \mathrm{dom} f,\ x_1 < x_2 \Rightarrow f(x_1) < f(x_2)$

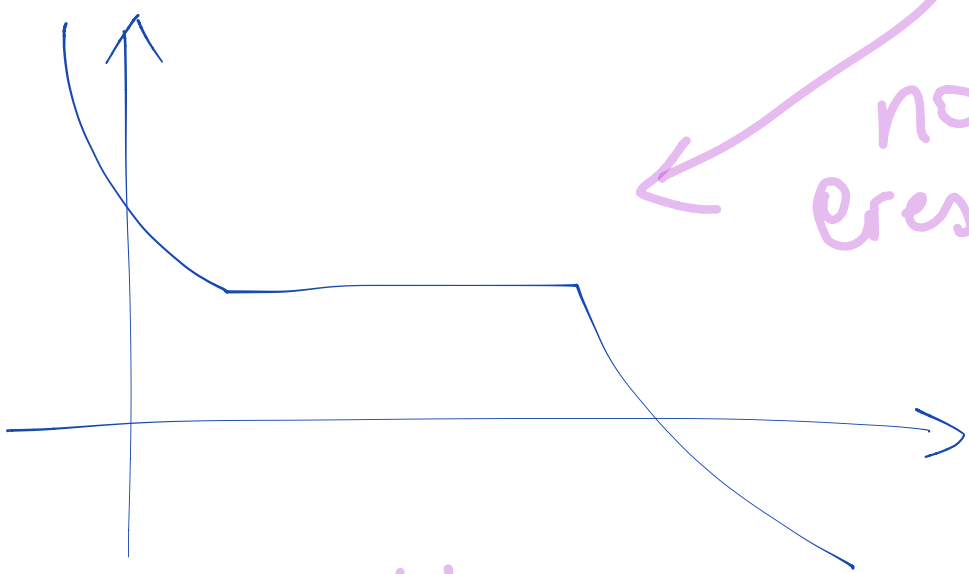


non decrescente



f è decrescente se

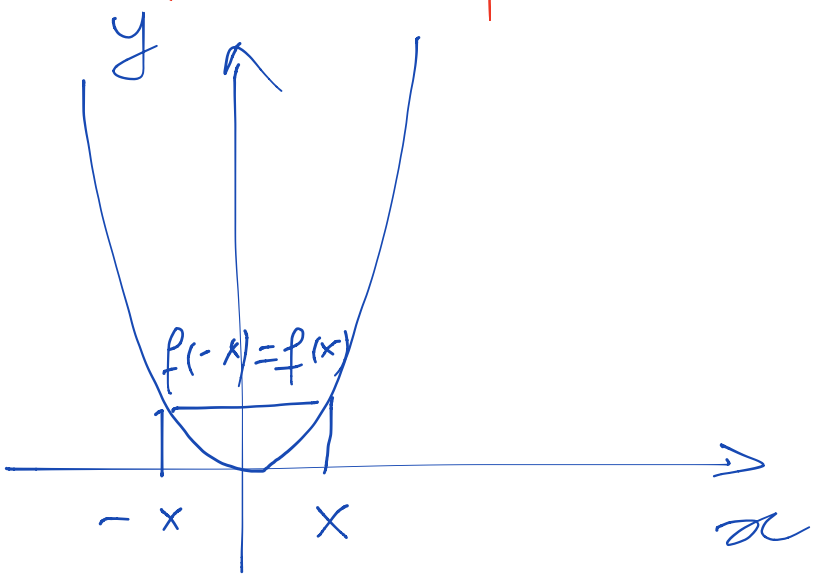
$\forall x_1, x_2 \in \text{dom} f,\ x_1 < x_2 \Rightarrow f(x_1) > f(x_2)$



non crescente

monotòne

Funzioni pari e dispari

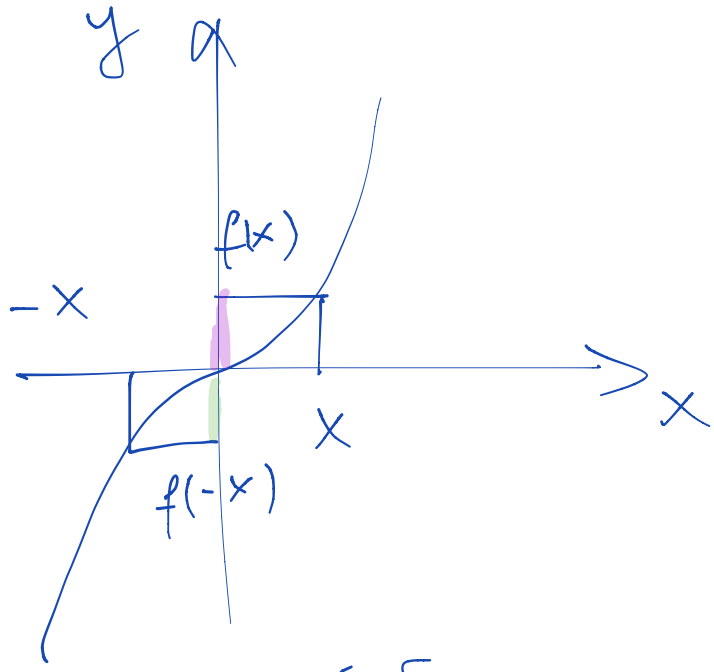


$$\forall x \in \mathrm{dom} f : -x \in \mathrm{dom} f \Rightarrow f(-x) = f(x)$$

$$y = \frac{x^2 - 1}{x^2 + 1}$$

$$f(-x) = \frac{(-x)^2 - 1}{(-x)^2 + 1} = \frac{x^2 - 1}{x^2 + 1} = f(x)$$

pari



f è dispari se

$\forall x \in \mathrm{dom} f : -x \in \mathrm{dom} f \Rightarrow f(-x) = -f(x)$

$y = x^3$

$f(-x) = (-x)^3 = -x^3$