$$1+x+x^2+x^3+\dots+x^n=\frac{1-x^{n+1}}{1-x}$$

$n\in\mathbb{N}$

Base induttiva $n=0$

$$1=\frac{1-x}{1-x}=1$$

Supposta la proprietà vera per n, dimostriamola per $n+1$

Ip. induttiva: $1+x+x^2+\dots+x^n=\frac{1-x^{n+1}}{1-x}$

tesi: $1+x+x^2+\dots+x^n+x^{n+1}=\frac{1-x^{n+2}}{1-x}$

$$\frac{1-x^{n+1}}{1-x}+x^{n+1}=\frac{1-\cancel{x^{n+1}}+\cancel{x^{n+1}}-x^{n+2}}{1-x}=$$

$$=\frac{1-x^{n+2}}{1-x}$$

$$2+2^2+2^3+\dots+2^n = 2(2^n-1)$$

Base ind. $n=1$

$$2 = 2\cdot(2-1)$$

Ip. ind: $2+2^2+2^3+\dots+2^n = 2(2^n-1)$

Tesi: $2+2^2+2^3+\dots+2^n+2^{n+1} = 2(2^{n+1}-1)$

$$2(2^n-1)+2^{n+1} = 2^{n+1}-2+2^{n+1} =$$

$$= 2\cdot 2^{n+1}-2 = 2(2^{n+1}-1)$$

$$1+3+5+7+\cdots+19$$

$$S_{10} = 10\,\frac{1+19}{2} = 100$$

$$1+3+5+7+\cdots+(2n-1)$$

$$S_n = n\,\frac{\cancel{1}+2n-\cancel{1}}{2} = n^2$$