

Esercizio 1. Semplificare le seguenti espressioni.

a) $(3y^n - x^m)(x^{m+1} - 4y^{n+2}) - (3y^n - x^m)(x^{m+1} + 2y^{n+2}) - 6y^n(x^m y^2 + y^{n+2})$ [R. $-24y^{2n+2}$]

b) $\left(\frac{1}{3}x^2 - y\right)\{-(x-3y)(x+3y+1) + [3y(x-3y-1)+x] - x(2-x)\} + x\left(\frac{2}{3}x^2 - 2y + 3y^2\right)$ [R. $x^3 y$]

c) $(a^p b^q - 1)(a^{2p} b^{2q} + a^p b^q + 1) - 2a^p(a^p b^q + 4)\left(\frac{1}{2}a^p b^{2q} + 1\right) + 2a^p(2a^p b^{2q} + a^p b^q + 4)$ [R. -1]

d) $(2, \bar{3}a^2 b + 0, 3a^3)\left(\frac{3}{10}a^3 - \frac{7}{3}a^2 b\right) + \left[\frac{1}{2}a^2 b(a-b)(a^2 - 2ab) - \frac{3}{2}ab\left(\frac{2}{3}a^2 b^2 - a^3 b\right)\right] + -a^4(0, 09a^2 - 5, \bar{4}b^2)$ [R. $\frac{1}{2}a^5 b$]

e) $(-2a)^2 \left[\left(\frac{3}{4}a - \frac{1}{2}b\right)(a-b) - 2\left(-\frac{3}{8}a + \frac{1}{4}b - 3ab\right)(2b-a) - (b-1)\left(12ab + 12a - 6a^2 - \frac{1}{2}b\right) \right] + -2a^2 \left(\frac{3}{2}ab + 24a - b \right)$ [R. $-24a^4$]

f) $\left(-\frac{1}{3}a^3 b^2\right) \left[(a^4 - 3a^3 b + 4a^2) - (5a^4 + 4a^2 + 3a^3 b) \right] : (-2a^3 b)^2 + -\left(\frac{4}{3}a^3 b^2 + 2a^2 b^3\right) : \left[(-4ab - 3a^2 b) + a(2b + 3ab) \right]^2$ [R. 0]

g) $\left[\left(-\frac{x^5 y^2}{2} + \frac{4}{7} - \frac{x^4 y^2}{3}\right) \left(-\frac{2}{3}xy^2\right) + \left(\frac{3}{7}y - \frac{x^4 y^2}{6} + \frac{5x^5 y^3}{6}\right) \left(\frac{3}{2}xy\right) \right] : \left(\frac{1}{6}xy^2\right) - 3x^4 y^2 \left(\frac{5}{2}x + \frac{5}{18}\right) + 2x^3 \left(\frac{1}{2}xy^2 - x^2 y^2\right)$ [R. $\frac{11}{7}$]

h) $\left[(0, 2m-0, 3n-1, 2)(5m-3n+1) - 3n(0, 3n-0, 7m-1, 1)\right] : \left[-2m\left(-\frac{1}{4}mn\right)^2\right] + -\frac{1}{2}m^2 n^2 \left(\frac{1}{2}m^6 - 3m^5 - \frac{3}{5}m^4\right) : (2m^3)$ [R. $\frac{1}{40}m^4 n^2 + \frac{33}{40}m^3 n^3$]

i) $\left[2(2a-b^2)^2 - (2a-3b^2)^2 - (2a-b^2)(2a+b^2)\right] \cdot \left[(b^2+2)^2 - (b^2-2)^2\right] + 48b^6$

j) $\left[(x^p - y^q)^3 + 3y^q(x^p + y^q)^2 - 9x^p y^{2q}\right] \cdot (x^{3p} - 2y^{3q}) + 4y^{6q}$

k) $6x\left(\frac{1}{2}xy\right)^2 \cdot \left[\left(\frac{1}{6}y+x\right)^2 - y\left(\frac{1}{36}y-2x^2\right)\right] : \left(\frac{3}{2}x^2y^2\right) - (x-y)^3 + y^2(3x-y)$

l) $\frac{3}{8}b \cdot \left\{ \left[(a-1)^2 - (a+1)^2\right]^3 + \left[(a+b)^2 - (a-b)^2\right]^3 \right\} : \left[(a+b)^3 - (a-b)^3 - 2b^3\right]$

m) $\left\{ \left(\frac{1}{3}a^2 + \frac{1}{2}b\right)^3 - \left(\frac{1}{3}a^2 - \frac{1}{2}b\right)^3 - \left[\left(\frac{1}{4}b^2 - 2a\right) + 4a\left(\frac{1}{3}a^3b + \frac{1}{2}b\right) + \left(-\frac{5}{3}a^2b\right) \right] \right\} : \left(-\frac{5}{3}a^2b\right)$

n) $\left[\left(\frac{1}{3}x - \frac{1}{2}y\right)^2 - \frac{1}{9}x(x-3y) + \left(-\frac{1}{2}y\right)^2 \right] : \left[\left(x - \frac{1}{2}y\right)^2 - x(x-y) + (-2x)^2 : (4x) \right]$

Esercizio 2. Sviluppare i seguenti quadrati di binomio.

a) $(x-5y)^2$

b) $\left(-\frac{1}{2}x^2 + 6y\right)^2$

c) $\left(-\frac{5}{2}xy + y^2\right)^2$

d) $(0, \bar{2}x^2 + 3y)^2$

e) $\left(0, \bar{3}x^2y - \frac{3}{2}y^3\right)^2$

f) $(x^{2p} - 3y^{p-1})^2$

g) $\left(x^m y^n - \frac{1}{3}x^{2m}y\right)^2$

h) $\left(\frac{3}{4}x^m y^n - \frac{8}{3}x^{2m}y^n\right)^2$

i) $\left(\frac{1}{2}x^{2m-1}y^{2n} + \frac{2}{3}x^m y^{3n-1}\right)^2$

j) $\left(x^p - \frac{3}{4}x^{5-p}\right)^2 \quad p \in \quad , p < 5 \quad k) \quad (a^{q+1} + 3a^{q-4})^2 \quad q \in \quad , q > 4 \quad l) \quad (0,01z - 0,1yz)^2$

Esercizio 3. Semplificare le seguenti espressioni.

a) $(a+1)^2 + a(a+1) - 2(a-1)^2 \quad 7a-1$

b) $4x\left(x - \frac{1}{2}y\right)^2 - 2x\left(y + \frac{1}{2}\right)^2 - \frac{1}{2}x(8x^2 - 4y - 1) \quad -4x^2y - xy^2$

c) $3x^2\left(\frac{1}{3}x - \frac{1}{2}y\right)^2 - \frac{1}{3}y^2\left(\frac{3}{2}x + y\right)^2 + y(x^2 + y^2)\left(\frac{1}{3}y + x\right) \quad \frac{1}{3}x^4 + \frac{1}{3}x^2y^2$

d) $x(x-10)^2 - 4(12x-1) + (x-2)(x+2) - x(x-4)(x+2) \quad -17x^2 + 60x$

e) $5a^4 - [(a+3b)(a-3b)]^2 + (-9b^2 - 2a^2)(-9b^2 + 2a^2) \quad 18a^2b^2$

Esercizio 4. Sviluppare i seguenti prodotti notevoli.

a)	$(p^m + q^n - r^m)^2$	$p^{2m} + q^{2n} + r^{2m} + 2p^m q^n - 2p^m r^m - 2q^n r^m$
b)	$(-2x^{m+1} - 3y^{n+1} - x^m y^n)^2$	$4x^{2m+2} + 9y^{2n+2} + x^{2m} y^{2n} + 6x^m y^{2n+1} + 12x^{m+1} y^{n+1} + 4x^{2m+1} y^n$
c)	$[a^3 - (b^3 + c^3)]^2$	$a^6 + b^6 + c^6 - 2a^3 b^3 - 2a^3 c^3 + 2b^3 c^3$
d)	$[2x - (x^2 + y)]^2$	$x^4 - 4x^3 + 2x^2 y + 4x^2 - 4xy + y^2$
e)	$[(a - 2b) - ab]^2$	$a^2 b^2 - 2a^2 b + a^2 + 4ab^2 - 4ab + 4b^2$
f)	$[(x^2 + 5y^2) + 3xy]^2$	$x^4 + 6x^3 y + 19x^2 y^2 + 30xy^3 + 25y^4$
g)	$[(a + b) - (c + d)]^2$	$a^2 + b^2 + c^2 + d^2 + 2ab - 2ac - 2ad - 2bc - 2bd + 2cd$
h)	$[(2x - 1) + (y + 4)]^2$	$4x^2 + 4xy + 12x + y^2 + 6y + 9$
i)	$\left(\frac{1}{2}ab - 1\right)\left(\frac{1}{2}ab + 1\right)\left(\frac{1}{4}a^2 b^2 - 1\right)$	$\frac{1}{16}a^4 b^4 - \frac{1}{2}a^2 b^2 + 1$
j)	$\left(\frac{3}{2}x^2 y - z\right)\left(\frac{3}{2}x^2 y + z\right)\left(\frac{9}{4}x^4 y^2 - z^2\right)$	$\frac{81}{16}x^8 y^4 - \frac{9}{2}x^4 y^2 z^2 + z^4$
k)	$(a+b-c)(a+b+c)$	$a^2 + 2ab + b^2 - c^2$
l)	$(4a - 2b - 3c)(4a - 2b + 3c)$	$16a^2 - 16ab + 4b^2 - 9c^2$
m)	$(a^2 - b^2 - c^2)(a^2 + b^2 + c^2)$	$a^4 - b^4 - c^4 - 2b^2 c^2$
n)	$\left(\frac{1}{2}a^2 - \frac{1}{3}b^2 - 1\right)\left(\frac{1}{3}b^2 - 1 + \frac{1}{2}a^2\right)$	$\frac{1}{4}a^4 - a^2 - \frac{1}{9}b^4 + 1$
o)	$\left(\frac{1}{5}a^2 - 3b + c^3\right)\left(c^3 - 3b - \frac{1}{5}a^2\right)$	$c^6 - 6bc^3 - \frac{1}{25}a^4 + 9b^2$
p)	$(x + y + z + t)(x + y - z - t)$	$-t^2 - 2zt + x^2 + y^2 - z^2 + 2xy$
q)	$(a + 2b + 3c + d)(a - 2b + 3c - d)$	$a^6 + 6ac - 4b^2 + 9c^2 - d^2 - 4bd$
r)	$\left(a - 2b - \frac{1}{2}c + 1\right)\left(a + 2b - \frac{1}{2}c - 1\right)$	$a^2 - ac - 4b^2 + \frac{1}{4}c^2 + 4b - 1$
s)	$(2x - a + b - c)(2x + a - b + c)$	$-a^2 + 2ab - 2ac - b^2 - c^2 + 4x^2 + 2bc$
t)	$(3a - 2b + c - 1)(3a + 2b - c - 1)$	$9a^2 - 6a - 4b^2 - c^2 + 4bc + 1$
u)	$(0, \bar{3}a^2 - 0, 5b^3)^3$	$\frac{1}{27}a^6 - \frac{1}{6}a^4 b^3 + \frac{1}{4}a^2 b^6 - \frac{1}{8}b^9$
v)	$(1, 5x^2 + 1, \bar{3}y^3)^3$	$\frac{27}{8}x^6 + 9x^4 y^3 + 8x^2 y^6 + \frac{64}{27}y^9$
w)	$(0, 5x^4 - 0, 2y^2)^3$	$\frac{1}{8}x^{12} - \frac{3}{20}x^8 y^2 + \frac{3}{50}x^4 y^4 - \frac{1}{125}y^6$
x)	$\left(2x^2 y^3 + \frac{1}{2}x^3 y^2\right)^3$	$\frac{1}{8}x^9 y^6 + \frac{3}{2}x^8 y^7 + 6x^7 y^8 + 8x^6 y^9$

Esercizio 5. Semplificare le seguenti espressioni.

a) $\left(\frac{2}{3}a - \frac{3}{2}b\right)^3 + \frac{3}{8}b \cdot \left\{ \left[\left(\frac{2}{3}b - \frac{3}{2}\right)\left(\frac{3}{2} + \frac{2}{3}b\right) - \left(\frac{4}{9}b^2 + \frac{3}{4}\right) \right] b \right\}^2 - \frac{1}{2}ab(9b - 4a)$ $\left[R. \frac{8}{27}a^3\right]$

b) $\left[\left(\frac{1}{2}a - b\right)^2 - \frac{1}{4}a^2\right]^3 - \left[\left(a + \frac{1}{2}b\right)^2 - \frac{1}{4}b^2\right]^3 + (a^3 - b^3)(a^3 + b^3) + 3a^2b^2(a - b)(a + b)$
 $\left[R. -3a^5b - 2a^3b^3 - 3ab^5\right]$

c) $(a - b)^3(a + b)^3 - \left[(a - b + 1)(a - b - 1) + 4\left(\frac{1}{2}ab + \frac{1}{4}\right)\right]^3 + 2b^2(b^2 - a^2)(b^2 + a^2)$ $\left[R. -8a^4b^2\right]$

d) $-\left\{-\left[x^2(x+1)(x^2 - x + 1) - x^2(x^3 + x - 1)\right]^3 + x^6(2+x)^3\right\} : (-2x^2)^3$ $\left[R. \frac{1}{4}x^3 + 3x\right]$

e) $(a^2 - ab)^4 : \left\{ -\left[(4a - b)(4a + b) - b^2\right] \cdot \frac{1}{8}a^2 \right\} - \frac{1}{2}(a^2 - b^2)(a^2 + b^2) - 2ab(a - b)^2$ $\left[R. a^2b^2 - a^4\right]$

f) $\left\{ \left[(b+1)^4 - (b-1)^4 \right] : (8b) \right\} (b^2 - 1)(b^4 + 1) - b^8$ $\left[R. -1\right]$

g) $\left\{ \left[\left(\frac{11}{3}a - 2 \right) \left(\frac{3}{2}a + b \right) - \left(\frac{3}{2}a + b \right) (2a - 1) - \left(5a + \frac{10}{3}b \right) \left(\frac{1}{2}a - 1 \right) \right] \left(\frac{1}{7}a \right) \right\}^4 - \frac{1}{6}a^6b(a + b)$
 $\left[R. \frac{1}{16}a^8 + \frac{2}{27}a^5b^3 + \frac{1}{81}a^4b^4\right]$