

Equazioni lineari

$$ax = 3 \quad a \in \mathbb{R}$$

$$a=1 \rightarrow x = 3$$

$$a=2 \rightarrow 2x = 3$$

$$x = 3 \Rightarrow \underline{3a = 3}$$

$$\Rightarrow 2bx = 2b - b^2$$

$$2bx = b(2-b)$$

a) Se $b = 0$, si ha:

$$0 = 0 \quad S = \mathbb{R}$$

b) Se $b \neq 0$, allora:

$$x = \frac{b(2-b)}{2b} = \frac{2-b}{2}$$

$$2) (b^2 - b)x = b - 1$$

$$b(b-1)x = b-1$$

a) Se $b \neq 0$ e $b \neq 1$, se ha:

$$x = \frac{b-1}{b(b-1)} = \frac{1}{b}$$

b) Se $b = 0$, se ha:

$$0x = -1 \quad S = \emptyset$$

c) Se $b = 1$, se ha:

$$0x = 0 \quad S = \mathbb{R}$$

$$3) x + a + (a-1)^2 = a[4a - (a-2x)]$$

$$x + a + a^2 - 2a + 1 = 4a^2 - a^2 + 2ax$$

$$x - 2ax = 2a^2 + a - 1$$

$$x(1-2a) = (a+1)(2a-1)$$

$$\begin{array}{c|cc|c} & 2 & 1 & -1 \\ \hline -1 & & -2 & 1 \\ \hline & 2 & -1 & 1 \end{array}$$

a) Se $1-2a \neq 0$, cioè se $a \neq \frac{1}{2}$, si ha:

$$x = \frac{(a+1)(2a-1)^{-1}}{1-2a} = -a-1$$

b) Se $a = \frac{1}{2}$, si ha:

$$0x = 0 \quad S = \mathbb{R}$$

4) $(a-3)x = 2b-4$

a) Se $a = 3$, si ha:

$$0x = 2b-4$$

\searrow
Se $b = 2$
 $S = \mathbb{R}$

\nearrow
Se $b \neq 2$
 $S = \emptyset$

b) Se $a \neq 3$, si ha:

$$x = \frac{2b-4}{a-3}$$

- Se $a = 3 \wedge b = 2$ $S = \mathbb{R}$
- Se $a = 3 \wedge b \neq 2$ $S = \emptyset$
- Se $a \neq 3$ $S = \left\{ \frac{2b-4}{a-3} \right\}$

Diseguazioni letterali

1) $(3x-6)a \geq 3(ka-x)$

$$3ax - 6a \geq 6a - 3x$$

$$3ax + 3x \geq 6a + 6a$$

$$3(a+1)x \geq 12a$$

$$(a+1)x \geq 4a$$

a) Se $a = -1$, si ha:

$$0x \geq -4 \quad S = \mathbb{R}$$

b) Se $a+1 > 0$, cioè se $a > -1$, si ha

$$x \geq \frac{4a}{a+1}$$

c) Se $a < -1$, se ha:

$$x \leq \frac{4a}{a+1}$$

z) $x(b-2)(b+2) - 3(1-2b) + 3x < x(b^2-1) + x(2b-1)$

~~$b^2x - 4x - 3 + 6b + 3x < b^2x - x + 2bx - x$~~

$$x - 2bx < 3 - 6b$$

$$(1-2b)x < 3(1-2b)$$

a) Se $\underline{b = \frac{1}{2}}$, se ha:

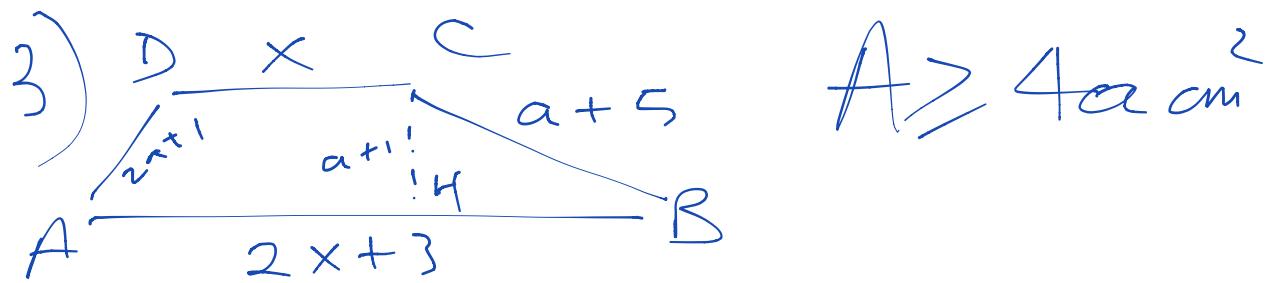
$$0x < 0 \quad S = \emptyset$$

b) Se $\boxed{1-2b > 0}$, cioè se $\underline{b < \frac{1}{2}}$, se ha:

$$x < \frac{3(1-2b)}{1-2b} \quad | \quad x < 3$$

c) Se $\boxed{1-2b < 0}$, cioè se $\underline{b > \frac{1}{2}}$, se ha:

$$\underline{x > 3}$$



$$A = \frac{(DC + AB) \cdot EH}{2} =$$

$$= \frac{(x + 2x + 3) \cdot (a + 1)}{2} =$$

$$= \frac{(3x + 3)(a + 1)}{2} \geq 4a$$

$$\frac{3ax + 3x + 3a + 3}{2} \geq 8a$$

$$\frac{3x(a+1)}{2} \geq 5a - 3$$

a) Non si discute il caso $a = -1$
perché non compatibile con le
condizioni geometriche

$$AD = 2(-1) + 1 = -1 \text{ imp.}$$

b) Per ragioni analoghe non si discute il caso $a < -1$.

c) Se $a > -1$, si ha:

$$x > \frac{5a-3}{3(a+1)}$$

$$\text{Se } A = 4a \Rightarrow x = \frac{5a-3}{3(a+1)}$$

$$\begin{aligned} P &= x + a + 5 + 2x + 3 + 2a + 1 = \\ &= 3x + 3a + 9 \end{aligned}$$

$$\begin{aligned} P\left(\frac{5a-3}{3(a+1)}\right) &= 3\left(\frac{5a-3}{3(a+1)}\right) + 3a + 9 = \\ &= \frac{5a-3}{a+1} + 3a + 9 = \frac{5a-3+3a^2+3a+9a+9}{a+1} = \\ &= \frac{3a^2+17a+6}{a+1} \end{aligned}$$