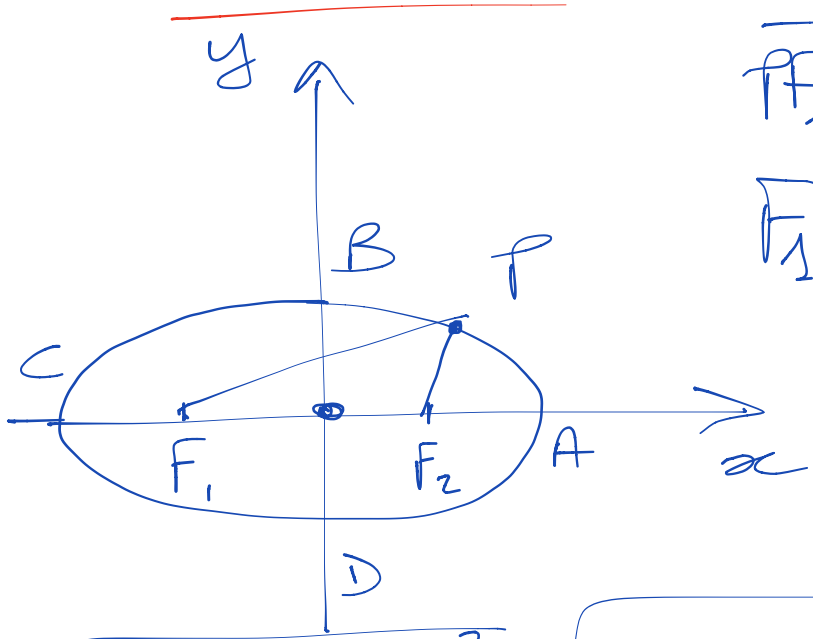
Ellisse



$$\overline{PF_1} + \overline{PF_2} = 2a$$

$$F_1(-c, 0)$$

$$F_2(c, 0)$$

$$\sqrt{(x+c)^2 + y^2} + \sqrt{(x-c)^2 + y^2} = 2a$$

$$\sqrt{x^2 + 2cx + c^2 + y^2} = 2a - \sqrt{x^2 - 2cx + c^2 + y^2}$$

$$\cancel{x^2} + 2cx + \cancel{c^2} + \cancel{y^2} = 4a^2 + \cancel{x^2} - 2cx + \cancel{c^2} + \cancel{y^2} - 4a\sqrt{x^2 - 2cx + c^2 + y^2}$$

$$4a\sqrt{x^2 - 2cx + c^2 + y^2} = 4a^2 - 4cx$$

$$a\sqrt{x^2 - 2cx + c^2 + y^2} = a^2 - cx$$

$$a^2x^2 - \cancel{2a^2cx} + a^2c^2 + a^2y^2 = a^4 + c^2x^2 - \cancel{2a^2cx}$$

$$(a^2-c^2)x^2+a^2y^2=a^2(a^2-c^2)$$

Poniamo $a^2-c^2=b^2$

$$b^2x^2+a^2y^2=a^2b^2$$

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \qquad \boxed{a>b}$$

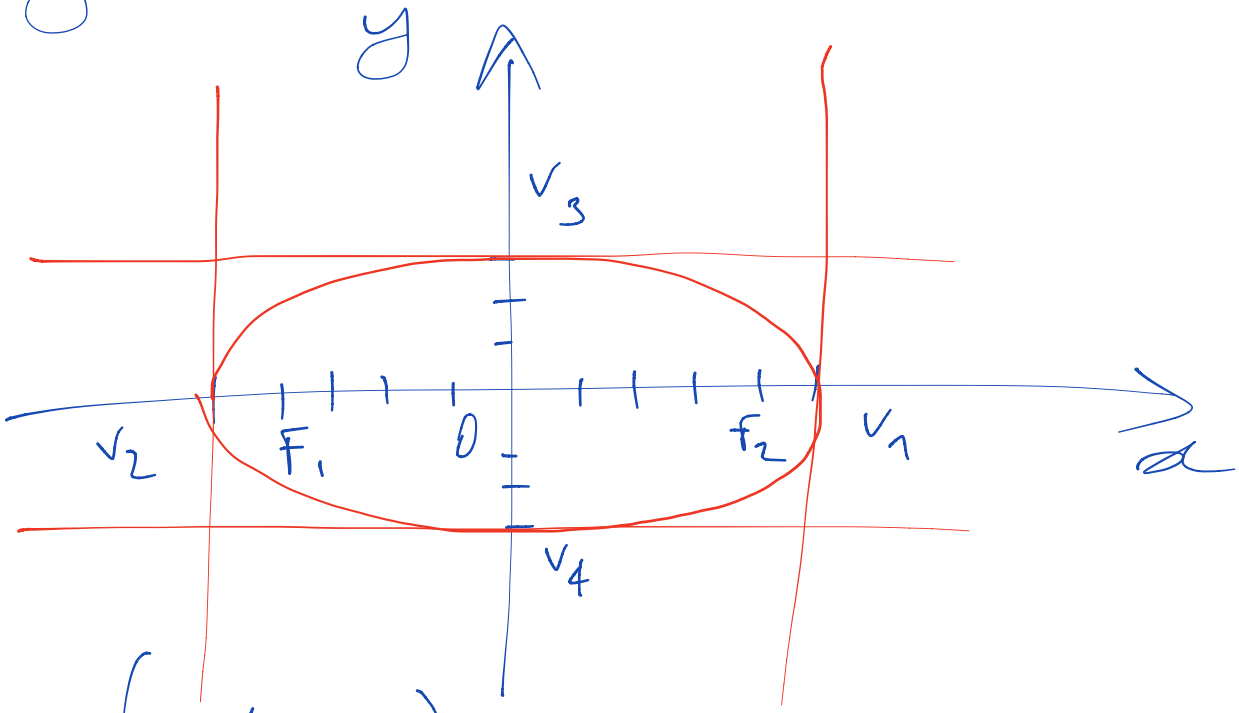
$$a^2-c^2=b^2 \Rightarrow c^2=a^2-b^2$$

$$F_1\left(-\sqrt{a^2-b^2};0\right) \quad F_2\left(\sqrt{a^2-b^2};0\right)$$

$$\frac{x^2}{25} + \frac{y^2}{9} = 1$$

$$x = 0 \Rightarrow y^2 = 9 \Rightarrow y = \pm 3$$

$$y = 0 \Rightarrow x^2 = 25 \Rightarrow x = \pm 5$$

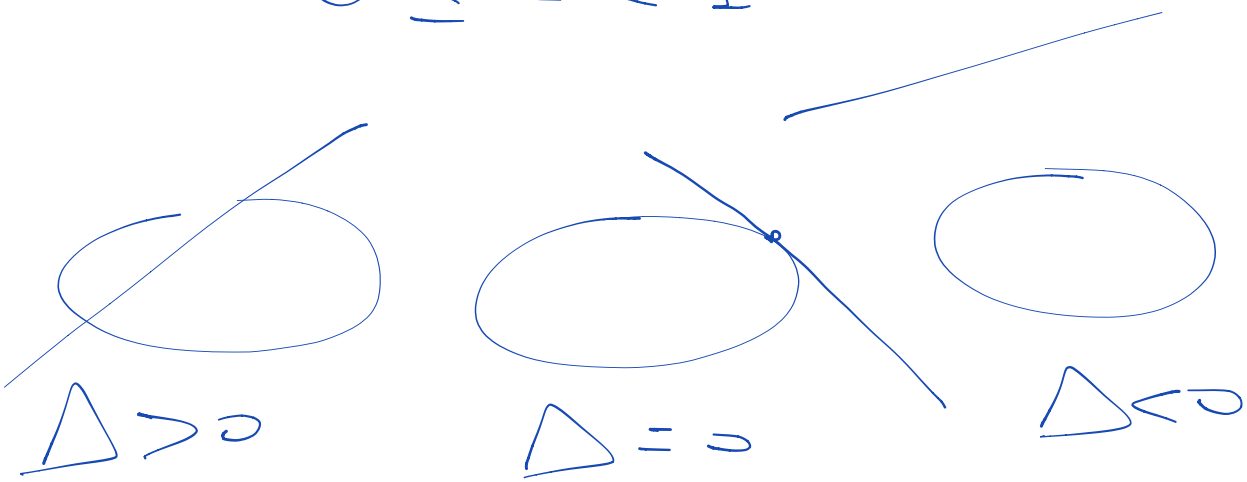


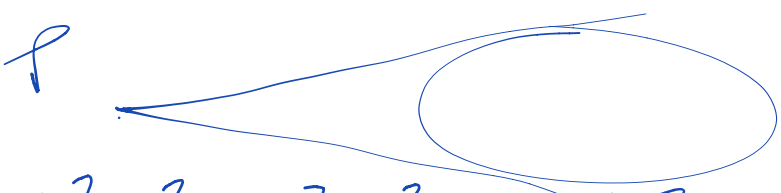
$$F(\pm 4; 0)$$

$$e = \frac{2c}{2a} = \frac{c}{a} = \frac{\sqrt{a^2 - b^2}}{a}$$

$e = 0 \Rightarrow c = 0 \Rightarrow$ circonf.

$0 \leq e < 1$





$$\begin{cases} b^2x^2 + a^2y^2 = a^2b^2 \\ y - y_0 = m(x - x_0) \end{cases} \qquad \Delta = 0$$

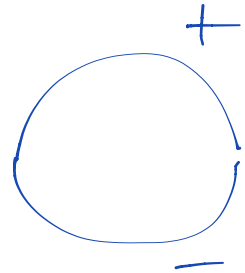
$P \in$ ellisse

$$\frac{x^2}{a^2} \longrightarrow \frac{x x_0}{a^2}$$

$$\frac{y^2}{b^2} \longrightarrow \frac{y y_0}{b^2}$$

$$x^2+y^2+ax+by+c=0$$

$$y^2+by+(x^2+ax+c)=0$$



$$b^2x^2+a^2y^2=a^2b^2$$

$$a^2y^2=a^2b^2-b^2x^2$$

$$y^2=\frac{b^2}{a^2}(a^2-x^2)$$

$$y=\pm\frac{b}{a}\sqrt{a^2-x^2}$$

$$a^2-x^2\geq 0 \qquad -a\leq x\leq a$$

$$\frac{x^2}{k+5} + \frac{y^2}{3k-1} = 1$$

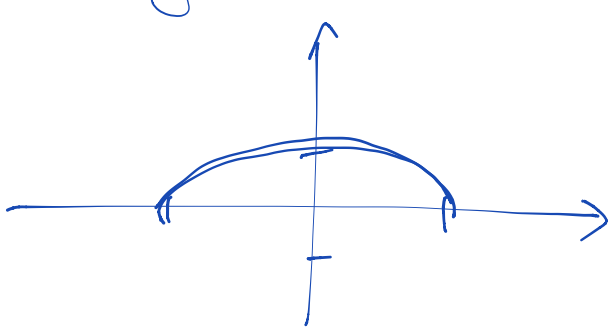
$$\begin{cases} 3k-1 > k+5 \\ k+5 > 0 \\ 3k-1 > 0 \end{cases} \qquad \begin{cases} k > 3 \\ k > -5 \\ k > 1/3 \end{cases}$$

$$y = \frac{1}{2}\sqrt{-x^2+1}$$

$$y^2 = \frac{1}{4}\left(-x^2+1\right)$$

$$4y^2 = -x^2+1$$

$$x^2 + 4y^2 = 1 \qquad x^2 + \frac{y^2}{\frac{1}{4}} = 1$$



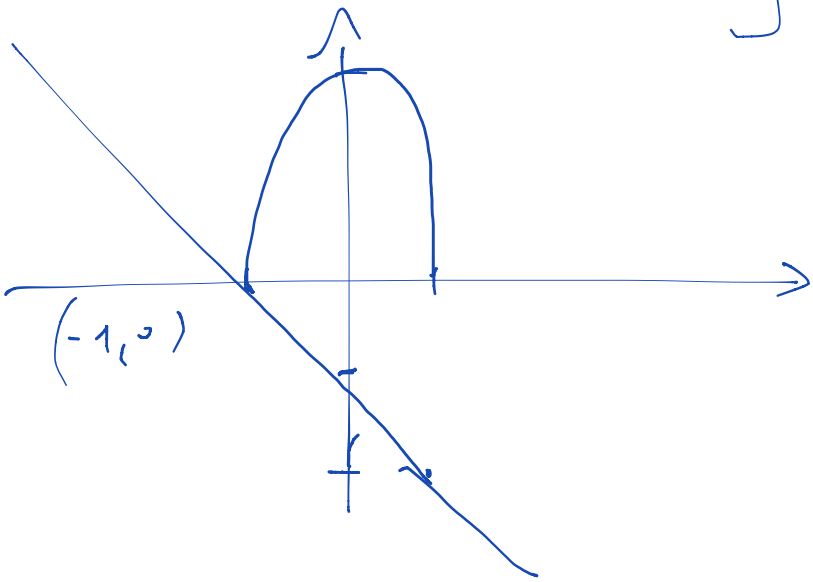
$$2\sqrt{1-x^2} \geq -x-1$$

$$y = 2\sqrt{1-x^2} \qquad y^2 = 4(1-x^2)$$

$$\frac{y^2}{4} = 1-x^2 \quad ; \quad x^2 + \frac{y^2}{4} = 1$$

$$y = -x-1$$

x	y
0	-1
1	-2



$$S = [-1;1]$$