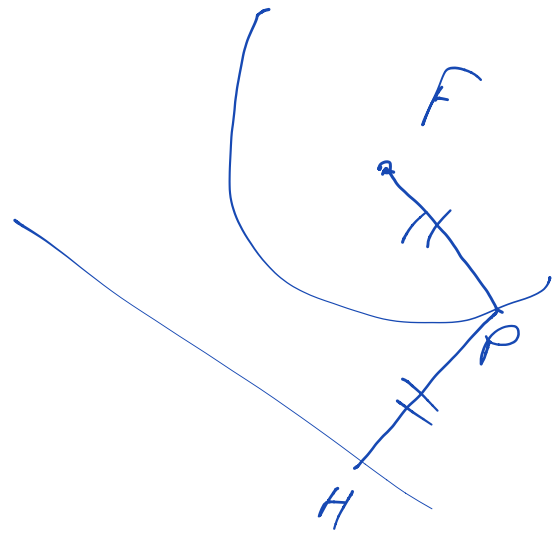
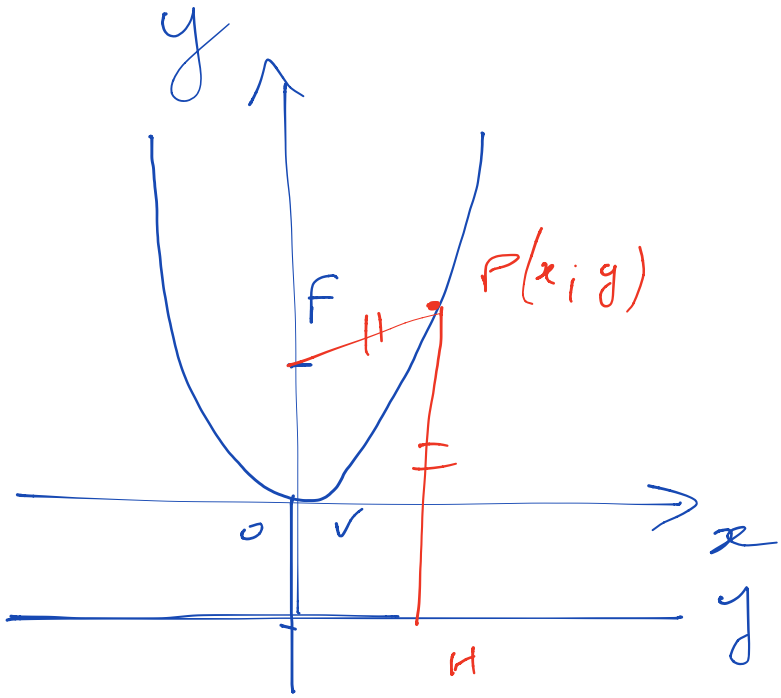
Parabole





$F(0; f)$

$y = -f$

$$\sqrt{(x-0)^2+(y-f)^2} = |y+f|$$

$$x^2+\cancel{y^2}-2fy+\cancel{f^2} = \cancel{y^2}+\cancel{f^2}+2fy$$

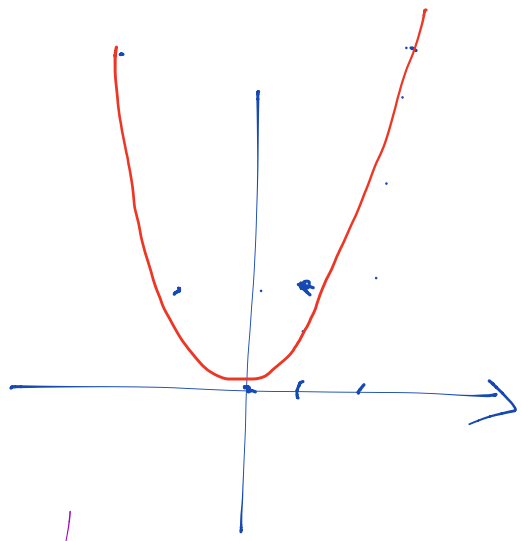
$$4fy = x^2 \Rightarrow y = \frac{1}{4f}x^2 \Rightarrow \boxed{y = ax^2}$$

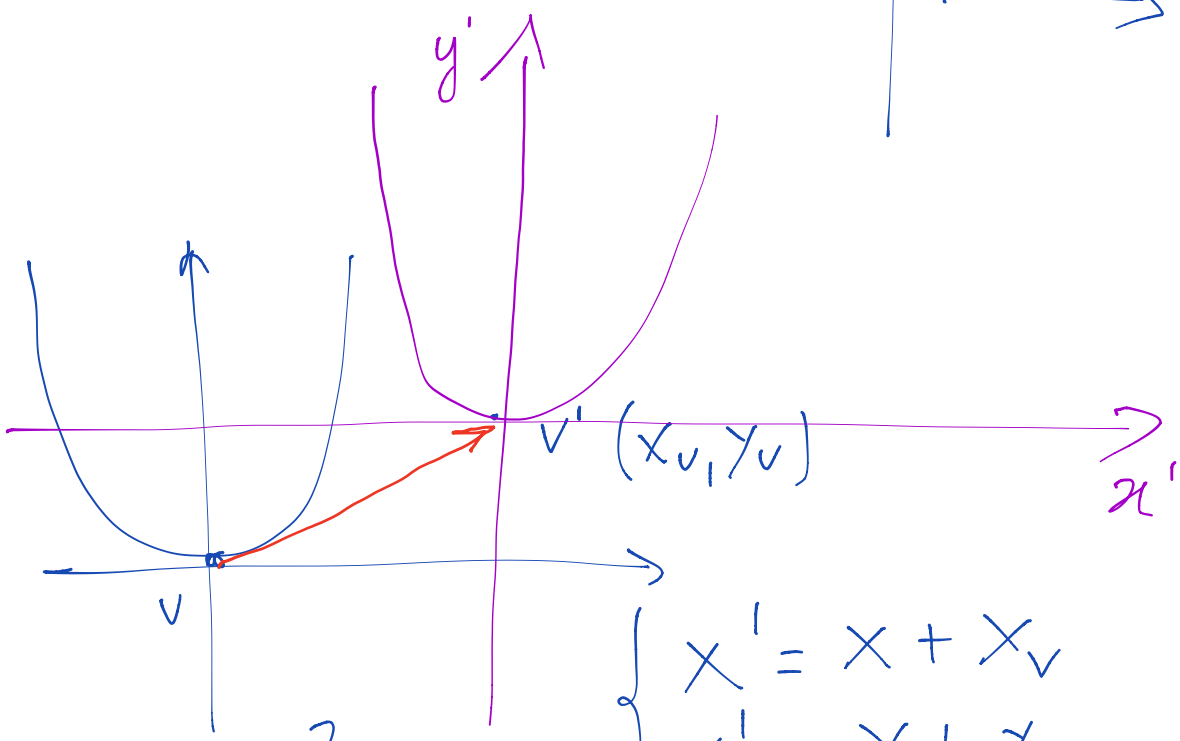
$$\frac{1}{4f} = a$$

$V(0,0)$ $\quad F\left(0;\frac{1}{4a}\right)$ $\quad y=-\frac{1}{4a}$ $\quad x=0$

$y=2x^2$

x	y
0	0
1	2
2	8





$$\begin{cases} x' = x + x_V \\ y' = y + y_V \end{cases}$$

$y=ax^2$

$$\begin{cases} x = x' - x_V \\ y = y' - y_V \end{cases}$$

$$y - y_v = a(x - x_v)^2$$

$$y - y_v = a(x^2 - 2xx_v + x_v^2)$$

$$y = ax^2 \underbrace{- 2ax_v}_{b} x + \underbrace{ax_v^2 + y_v}_{c}$$

$$y = ax^2 + bx + c$$

$$-2ax_v = b \Rightarrow x_v = -\frac{b}{2a}$$

$$ax_v^2 + y_v = c$$

$$y_v = c - ax_v^2 =$$

$$= c - a\left(-\frac{b}{2a}\right)^2 =$$

$$= c - \cancel{a} \cdot \frac{b^2}{4a^{\cancel{2}}} = \frac{4ac - b^2}{4a} = \frac{-\Delta}{4a}$$

$$V\left(-\frac{b}{2a}; -\frac{\Delta}{4a}\right)$$

$$F\left(0, \frac{1}{4a}\right) \longrightarrow F\left(x_V; \frac{1}{4a} + y_V\right)$$

$$F\left(-\frac{b}{2a}; \frac{1-\Delta}{4a}\right) \qquad x = -\frac{b}{2a}$$

$$y = -\frac{1}{4a} + y_V = -\frac{1}{4a} - \frac{\Delta}{4a} = \frac{-1-\Delta}{4a}$$

$$y = x^2 - 5x + 6$$

$$\Delta = 25 - 24 = 1$$

$$x_v = -\frac{b}{2a} = \frac{5}{2}$$

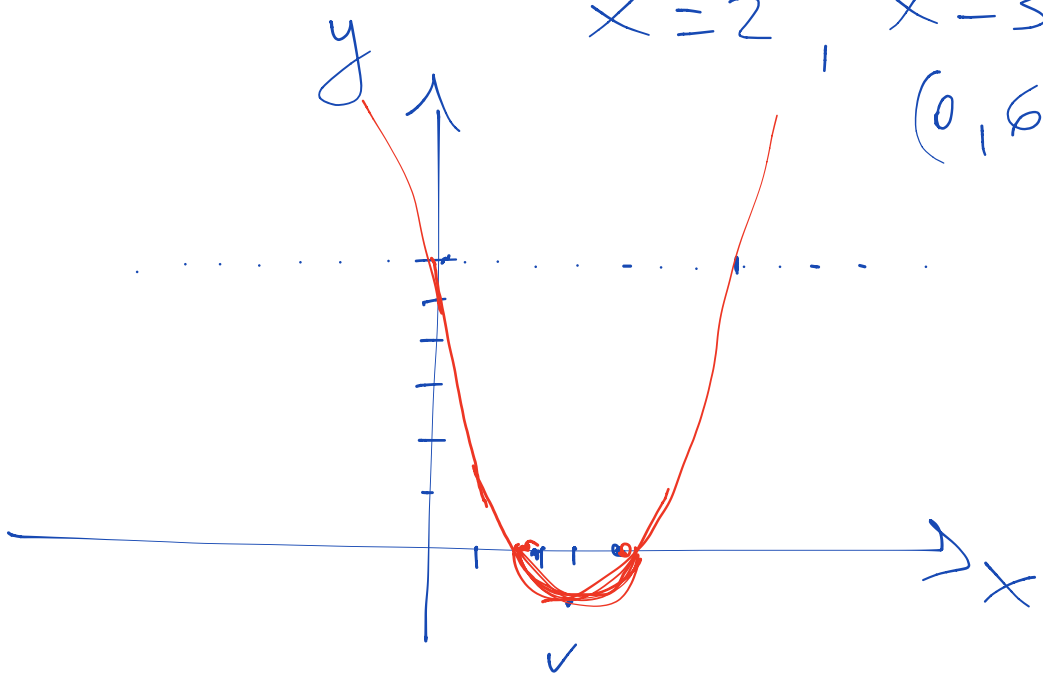
$$V\left(\frac{5}{2}, -\frac{1}{4}\right)$$

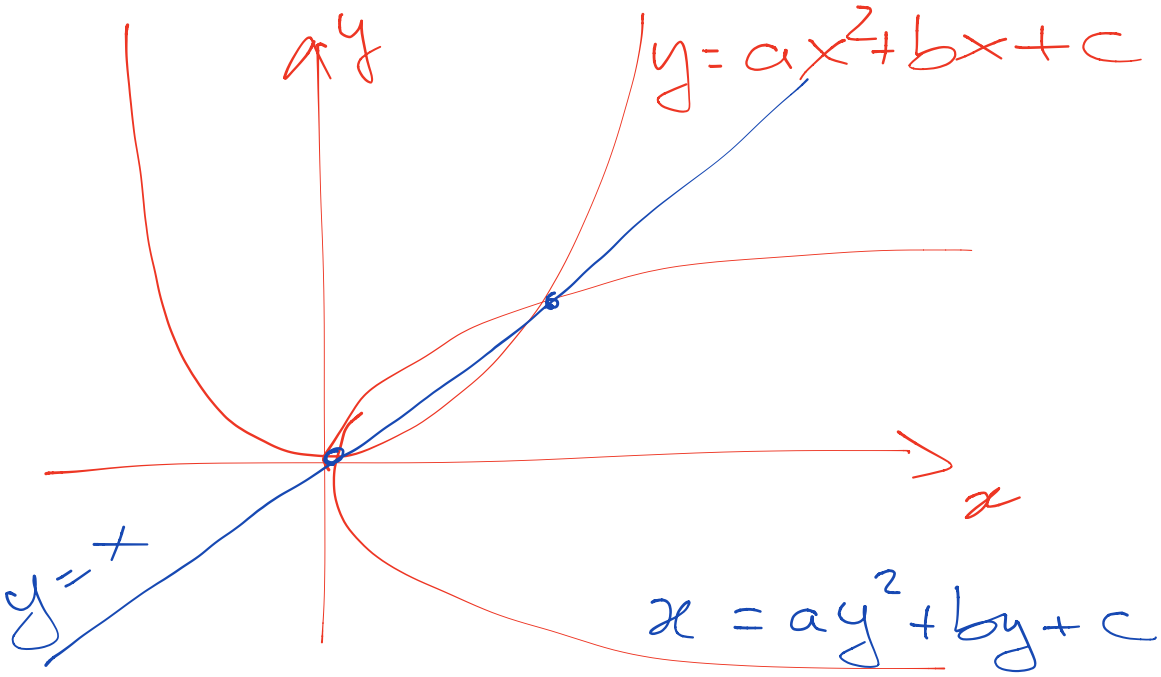
$$y_v = -\frac{\Delta}{4a} = -\frac{1}{4}$$

$$x^2 - 5x + 6 = 0 \qquad (x-2)(x-3) = 0$$

$$x = 2, \quad x = 3$$

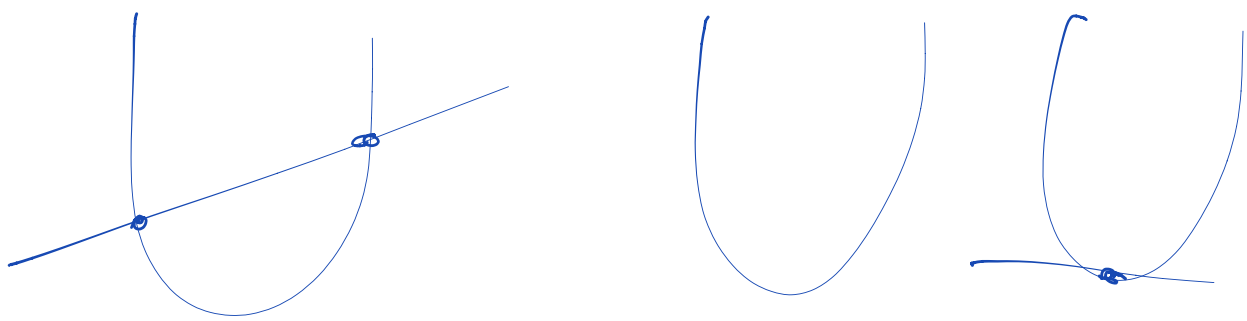
$$(0, 6)$$





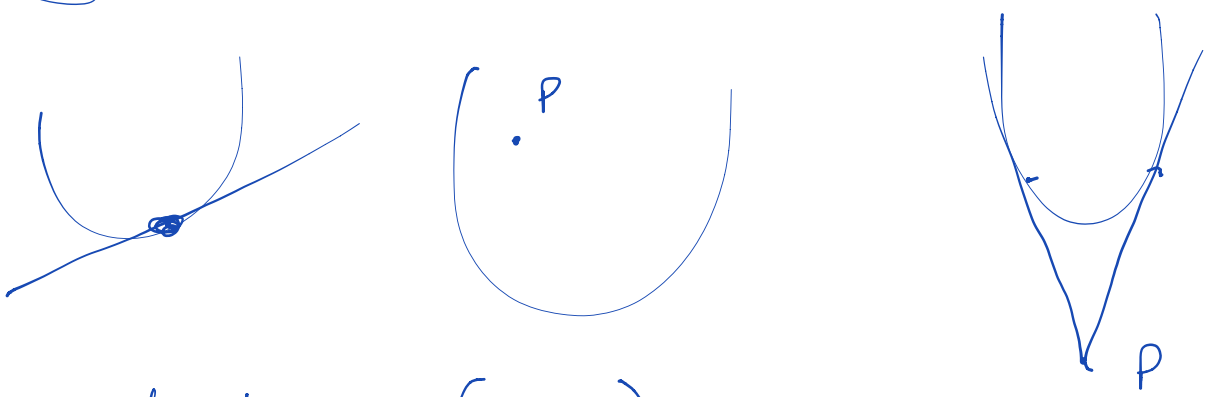
$$V\left(-\frac{\Delta}{4a}\,;\,-\frac{b}{2a}\right) \qquad F\left(\frac{1-\Delta}{4a}\,;\,0\right)$$

$$x = -\frac{1-\Delta}{4a} \qquad y = -\frac{b}{2a}$$



$\Delta > 0$ $\qquad$ $\Delta < 0$ $\qquad$ $\Delta = 0$

$y = 2x^2 - x - 1 \qquad P(1,1)$



$y - 1 = m(x-1)$

$$\begin{cases} y = 2x^2 - x - 1 \\ y = mx - m + 1 \end{cases}$$

$2x^2 - x - 1 = mx - m + 1$

$2x^2 - (1+m)x + m - 2 = 0$

$\Delta = 0$

$\Delta = (1+m)^2 - 4 \cdot 2(m-2) =$

$= 1 + 2m + m^2 - 8m + 16 = m^2 - 6m + 17 = 0$

$$\frac{\Delta}{4} = 9 - 17 < 0$$

$$y = ax^2 + bx + c \qquad P(x_0, y_0)$$

$$m = 2ax_0 + b$$

$$y - y_0 = (2ax_0 + b)(x - x_0)$$

se $P \in$ parabola

n. 350 $\quad y = ax^2 + bx + c$

$A(0,0) \quad B(1,2) \quad C(3,0)$

pass. per A: $c = 0$

pass. per B: $a + b + \cancel{c} = 2$

pass. per C: $9a + 3b + \cancel{c} = 0$

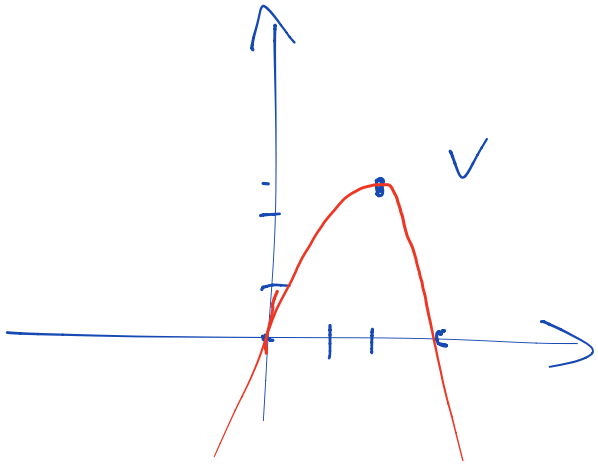
$$\begin{cases} c = 0 \\ a = 2 - b \\ 18 - 9b + 3b = 0 \end{cases} \qquad \begin{cases} c = 0 \\ b = 3 \\ a = -1 \end{cases}$$

$$y = -x^2 + 3x$$

$$x_V = -\frac{b}{2a} = \frac{-3}{-2} = \frac{3}{2} \qquad V\left(\frac{3}{2}; \frac{9}{4}\right)$$

$$y_V = -\frac{9}{4} + \frac{9}{2} = \frac{-9+18}{4} = \frac{9}{4}$$

$x = 0 \qquad x = 3$


V